Analysis of the curved lines on the image "Folien 1 unsealed"

When lifting fingerprints from a glass, the top and bottom edges of the glass could leave impressions on the print. I assume that in the transferal process the flexible recording media is wrapped smoothly around the glass, which is equivalent to rolling the glass smoothly (i.e. without slipping) on the flat recording medium. I consider the two cases of a cylindrical glass and a conical glass separately.

If the glass is cylindrical, then the top and bottom edges will trace out two parallel lines, and their separation (measured perpendicularly to the lines) will yield the height of the glass. Any straight line on a print could therefore be caused by one edge of a cylindrical glass. If another edge was captured at the same time, they must be parallel with respect to each other. It is also worth noting that I can say nothing about the diameter of this glass from the information on the print.

In order to discuss the case of a conical glass, I present an exaggerated conical glass in Figure 1. I define the apex of the cone as the point O. The distance between O and a point on the edge of the truncated base is R_2 while the distance between O and a point on the top edge is R_1. The circle at the base has radius r_2, while the circle at the top has radius r_1. The vertical height of the glass is h, and the length along its side is, $l = R_1 - R_2$. The half-angle of the cone (α) relates h to l by $h = l \cos \alpha$ and also $r_i = R_i \sin \alpha$, for both $i = 1,2$.

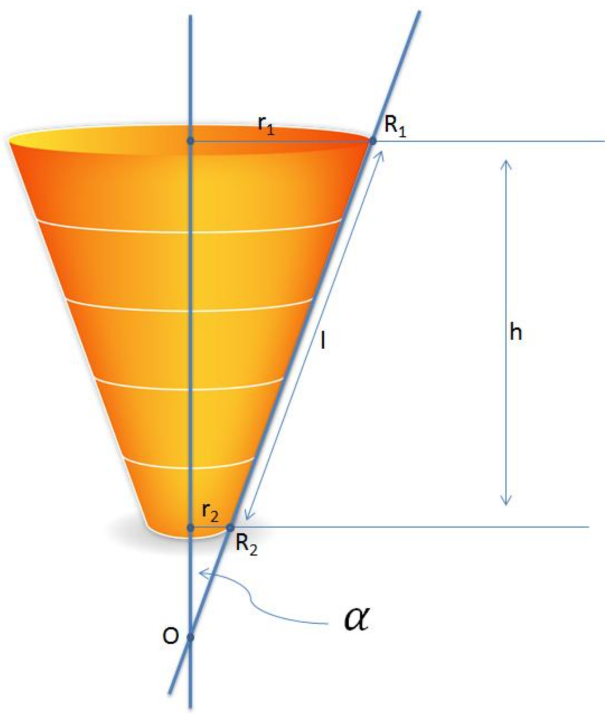


Figure 1: A truncated cone as a model for a conical glass.

If such a truncated cone is rolled onto a flat surface the top edge will trace out part of a circle (with origin at O) with radius R_1. The base will also trace out a circle with radius R_2 with the same origin O. These two circular arcs are concentric. The difference of their radii i.e. $R_1 - R_2$ will yield the side length l.

However, I cannot say anything about h nor r_1 or r_2, because I do not know what the value of the half-angle α is. To see this, imagine that I make a new cone by doubling α only. I now have a different sized truncated cone. R_1 and R_2 stay the same (therefore the side length l too) and will trace out the same size concentric circular arcs, but h will now be shorter (whereas both r_1 and r_2 will be larger). In general one cannot uniquely determine the shape of a conical glass from a print. However given a particular glass one can uniquely calculate the circular arcs that it should produce on a print, it is just that certain other types of glasses can also produce the same arcs.

While I cannot say anything about r_1 and r_2 separately, I can determine their ratio, given by $r_1/r_2 = R_1/R_2$. In this ratio the dependence on $\sin\alpha$ cancels out and thus R_1/R_2 determines the ratio of the top and bottom radius (or diameter or circumference).

The reasoning above therefore suggests that if one can determine the common origin and the radii of the arcs on the print, one can determine the side length and the ratio of the top and bottom diameters. This can be done by simultaneously fitting two concentric circles (in these cases I only care about the top half of the circles) to a set of data points. This is a four parameter fit on the data, that determines the coordinates of the origin namely x_0 and y_0, and the two radii R_1 and R_2.

Test of method

I first apply this method to a report[1] which contains a link to a photograph of a lift from a glass[2] as well as a table giving the dimensions of a number of glasses used in these tests. In Figure 2 I show

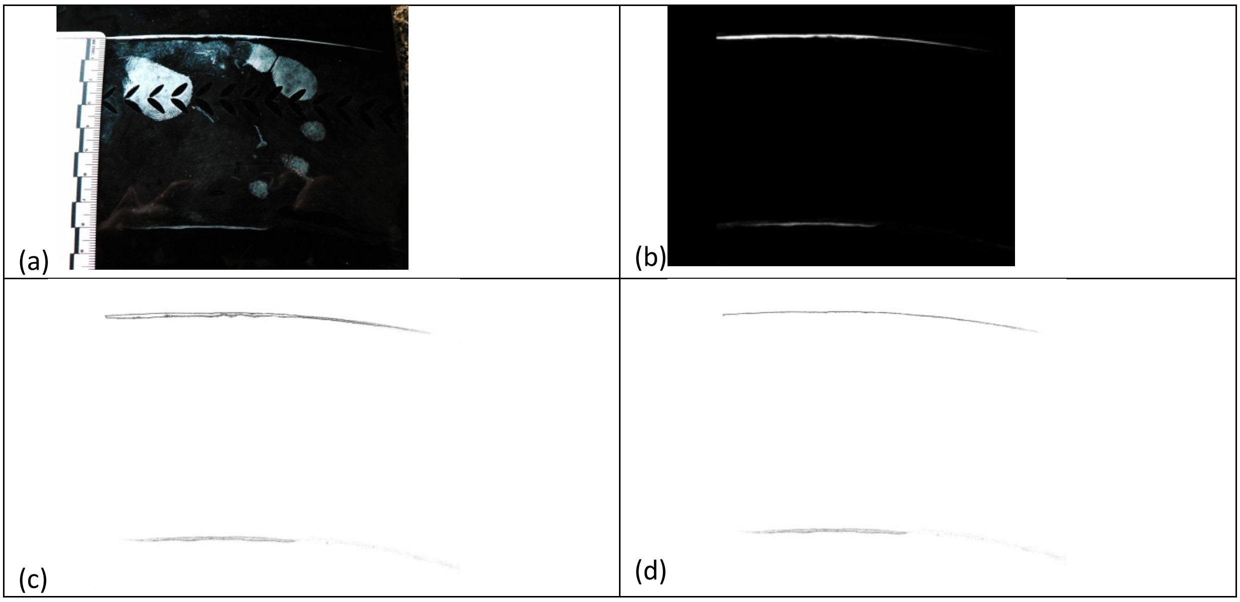


Figure 2: (a) image as downloaded. (b) Erasing all pixels except where edges are suspected. (c) Result of an edge-detection algorithm. (d) Figure after erasing bottom line of top edge that was used as data points to the fitting routine.

the steps that were taken to convert the image into pairs of (x,y) data points.

[1] http://www.clpex.com/Articles/VanDerVyver/VanDerVyverLPreport.htm
[2] http://www.clpex.com/Articles/VanDerVyver/LiftFromGlass-Original.jpg

Firstly those parts of the image that were not deemed part of the edges, were erased to black (b). Then an edge detection algorithm (IrfanView[3]) was run and the result is shown in (c). Here it was noticed that the bottom part of the top edge was highly irregular, and that was then erased, leaving only the top edge of the top arc, and leaving the bottom arc as before. The result is shown in (d). This figure was then converted to a data set, keeping only the non-white pixels (61,526 pixels). A non-linear least square fitting was performed in Maple 13[4] (it uses a library developed by the Numerical Analysis Group: NAG), using the colour of a pixel as a weight. Thus the darker pixels were weighted heavier than the lighter pixels. The results of that fitting allowed determination of the side length (80mm) and the ratio of the top to bottom diameter to be 1.22. This corresponds very well with the dimensions given in the report (see footnote 1) except that the side length is 2mm shorter than the values given in the report. The pattern of the glass used serves as corroboration that the correct glass has been identified.

Table 1: Dimensions of glasses together with the fitted results, indicating that the dimensions of glass #2 are consistent with Figure 2(a).

No	h [mm]	Top [mm]	Bottom [mm]	Ratio top/bot	Side length fit [mm]	Ratio-fitted
#1	82	74	67	1.10		
#2	82	79	64	1.23	80	1.22
#3	82	77	59	1.31		
#4	90	92	53	1.74		
#5	81	50	35	1.43		
#6	80	75	69	1.09		
#7	90	75	64	1.17		
#8	80	71	45	1.58		
#9	85	52	40	1.30		
#10	71	70	36	1.94		

Figure 3: Photograph of the glasses used in the report (footnote 1) with dimensions given in Table 1.

[3] http://www.irfanview.com/
[4] Maple 13.0; Maple Build ID 397624; Copyright (c) Maplesoft, a divsion of Waterloo Maple Inc, 1981-2009.

Figure 4: Result of fitting overlaid on original image.

The fitted results are shown on the original image in Figure 4. I have fitted two concentric circles. The top circle passes along the top edge of the imprint, while the bottom circle passes on average through the brighter markings associated with the bottom edge. The ruler shows that the distance between the two circles is just less than 82 mm, while our fitted results show 80 mm as the difference between the two radii. This is easily understood by noting that the ruler is not placed perpendicularly to the top edge. The lower part of the ruler should be rotated anti-clockwise around its origin (the 0 mm point. This will bring the actual side length measurement (using the ruler) into closer agreement with the fitted result. I note that the photograph of the lifted print has been taken at an angle (and not perpendicularly as would be the case when scanned in). I estimate that this does not affect the result significantly.

I have tried this fitting procedure using a more restrictive area for the bottom edge (which is not so well-defined as the top edge) without any significant differences noted.

Fitting Folien 1 unsealed

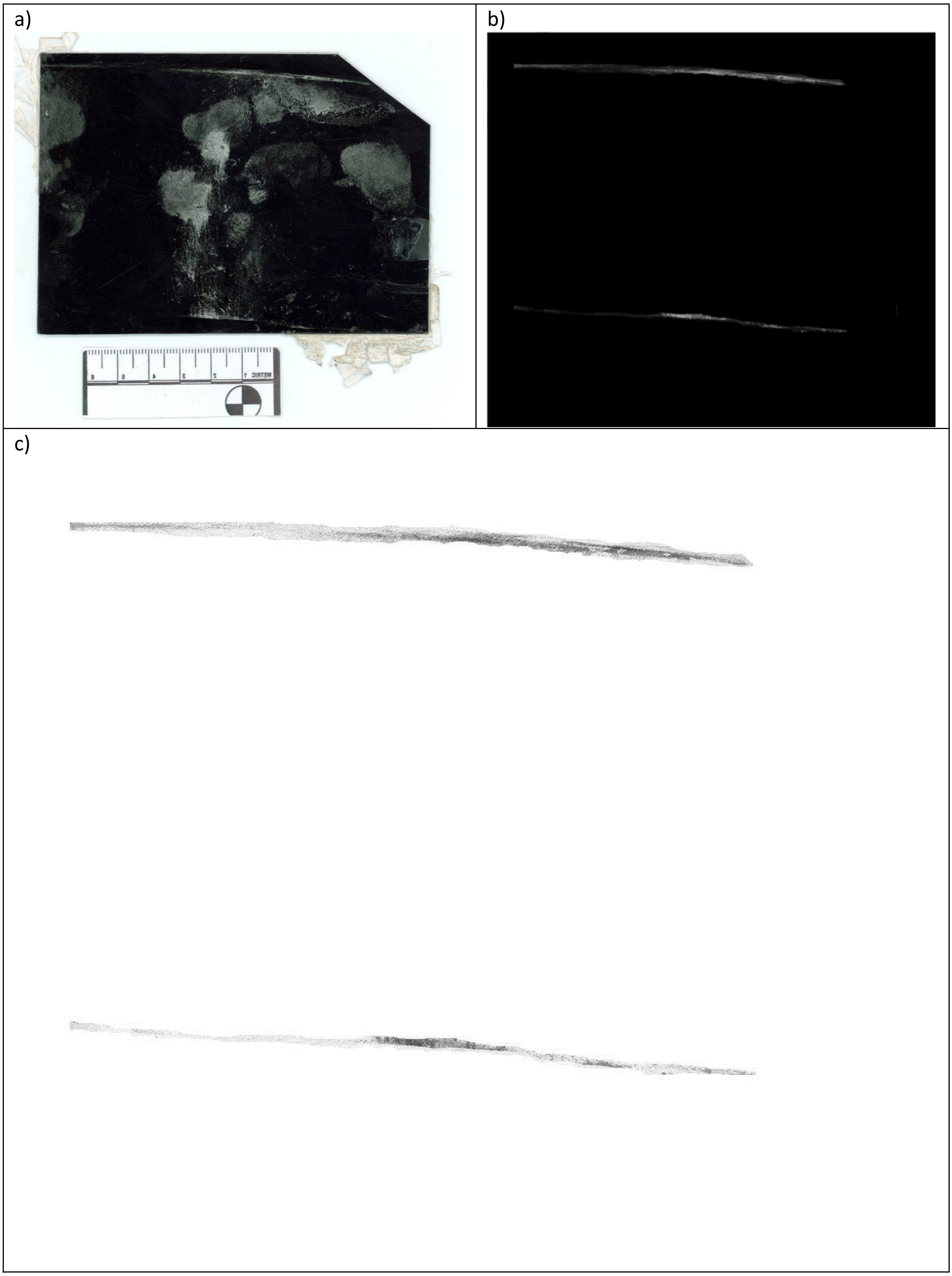


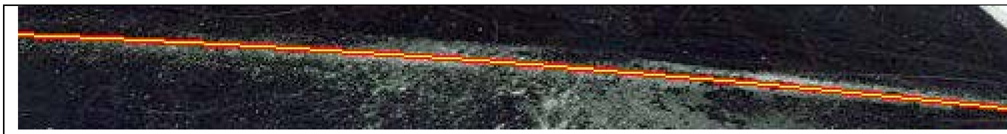
Figure 5: (a) Original image, (b) erased pixels not near edges (c) negative image used for fitting.

In Figure 5(a) I show the original image of Folien 1 (a scan made from the unsealed folien) and the processing required to get the data ready for fitting. The images were viewed in high magnification and those parts of the image that were not close to the perceived edges were erased (b). At this point I tried to do edge detection but the results were very noisy, so it was decided to simply use the negative and scale all the intensities to make maximum use of the intensity scale (c). These pixels (31,969) were then analysed as before. The best two concentric circles that can be fitted through the data of (c) is shown in Figure 6 below.

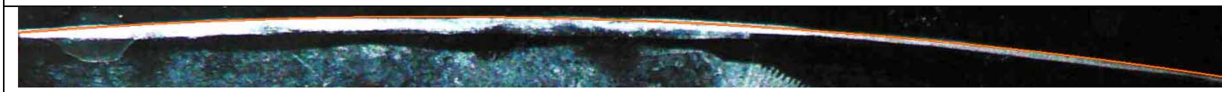
Figure 6: Fit of two concentric circles overlaid on original image of "Folien 1 unsealed".

As measured from the bottom left $(0{,}0)$ of the picture in pixels, the origin of both circles is at $(90, -6\ 340)$ and the top radius $R_1 = 7\ 276$ pixels, while the bottom radius is $R_2 = 6\ 639$ pixels. When converted to mm using the scale bar visible in the picture, this yields a side length of $l = 79$ mm and a ratio of top to bottom diameter of 1.10. It is interesting to compare this with Table 1 and note that glass #6 and possibly #1 are consistent with these lines drawn onto Figure 6. All the other glasses in that table can definitely be excluded.

It is, however, immediately obvious that this fit differs from that obtained from the model glass lift in Figure 4. I compare the quality of the fits in Figure 7.

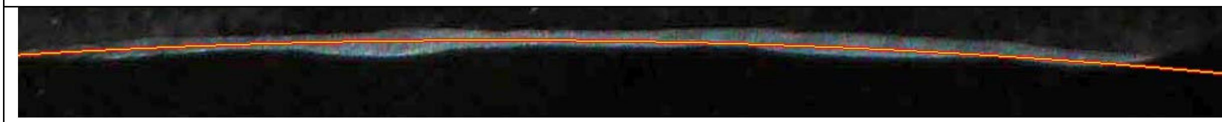


(a1) Detailed view of fit to top edge in Folien 1.



(a2) Detailed view of fit to top edge of glass #2.

(b1) Detailed view of fit to bottom edge in Folien 1.



(b2) Detailed view of fit to bottom edge of glass #2.

Figure 7: Comparison of the features fitted in Folien 1 with those of glass #2.

In comparison the top edge (see Figure 7(a1)) is much less defined than that of glass #2 (Figure 7(a2)), which displays a remarkable smoothness with a consistent curvature over a wide range of the picture. The edge in Figure 7 (a1) seems to be composed of a set of straight line segments. This will be discussed further in the next section. Comparing the bottom edges I note that in the image of glass #2 the bottom edge is not nearly as well defined as in its top edge. Although the edge width is not always the same thickness the fiitted line does pass through it fairly well. It might be argued that the top edge of Figure 7 (b2) might have been better used to fit the bottom edge, but that would only affect the measurement of the side length as it is clear that the top edge almost follows the fitted curve. In the case of Folien 1, Figure 7 (b1) shows no curve that follows the general shape of the top edge can fit the bottom edge, as there is the jagged bit at “x” that does not form part of the same curvature as the portion of the edge to its left.

Individual segments in the top edge of Folien 1

After recognizing that the top edge of Folien 1 could consist of a number of shorter line segments I tried to isolate these and do fitting on each on separately. I decided to divide the top edge into three line segments (called L1, L2 and L3). In Figure 8(b-d) I show the definition of these three line segments, obtained by isolating the regions out of the image used in (a). The results of fitting these three segments is overlaid in Figure 8(e). In each case I fitted a circular arc to the data, and then also fitted a straight line to the same data.

In the case of segment L1, I were not able to get a stable fitting solution for a circular arc, indicating that the data as very well described by a straight line. In the case of segment L2 and L3 I were able to fit circular arcs to the data.

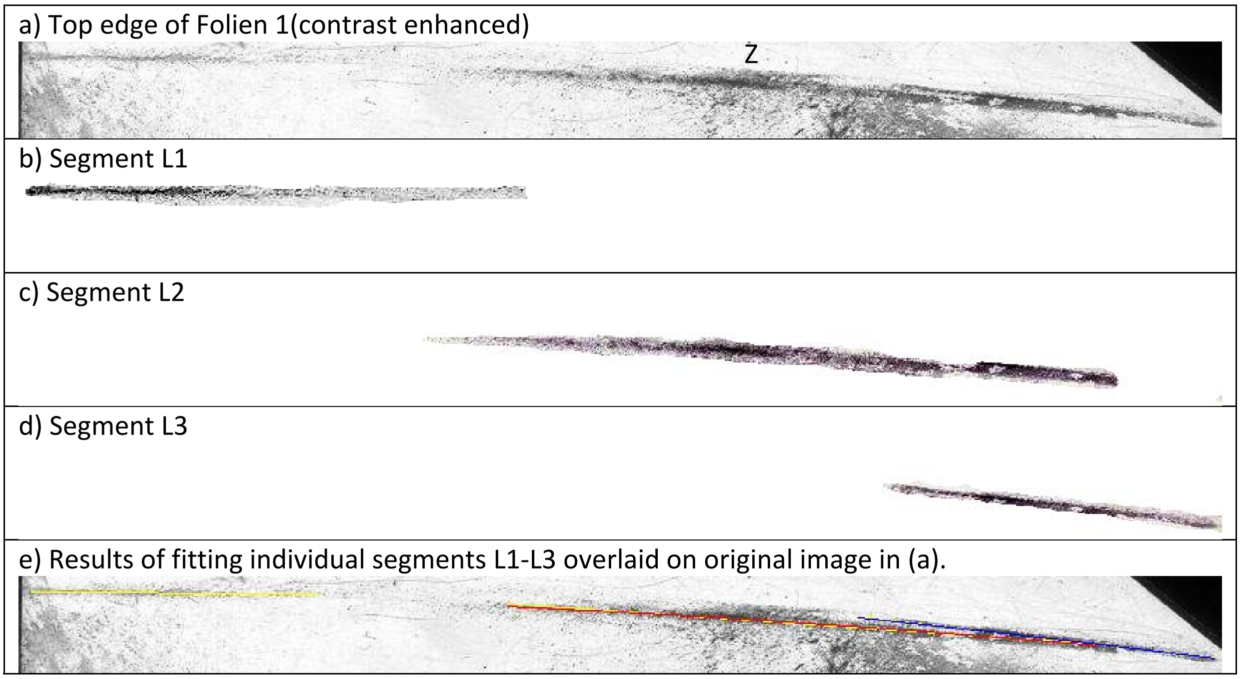


Figure 8: Fitting of three line segments isolated from the top edge of Folien 1.

In Figure 8(e) the straight line fits are drawn in yellow, while the fitted arc for L2 is shown inn red and that for L3 in blue. The radius of the arcs are much large than the value of R_1. For L2 and L3 they are 1.5 and 1.7 larger than $R_1 = 7\ 276$ pixels with separate origins more than 2000 pixels apart. There is also a further straight line segment (labelled "Z" in Figure 8a) that I have not analyzed. It is absolutely clear that these four segments could not have been made from the same conical glass.

In Figure 9 I show the last image (e) in greater detail. The yellow lines were drawn first and are the straight line fits. The red and blue were then drawn on top of that and are the result of fitting a circular arc to the data.

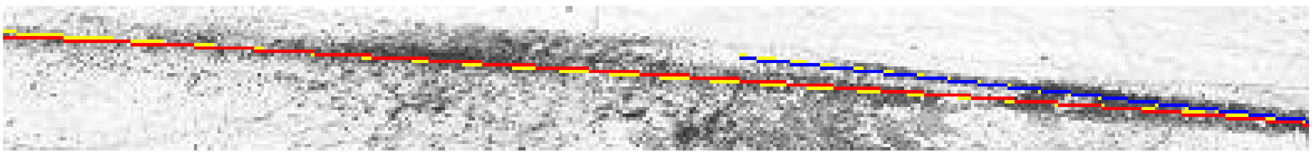


Figure 9: Magnification of Figure 8(e).

Conclusions

I have used nonlinear least square fitting techniques to determine parameters of conical glasses that would match traces of their edges left on a lifted imprint of the side of the glass. I accurately identified the parameters for glass #2 using the technique. On the assumption that the edges produced in Folien 1 come from a conical glass, I determine that a glass with dimensions similar to glass #6 will produce arcs that roughly match the two edges. (Other glasses could be consistent too, but they should have a side length of 79 mm and the ratio of top to bottom diameters must be 1.10). However, this fit is not satisfactory and cannot account for features in the bottom edge nor the top edge of Folien 1. In fact, the top edge in Folien 1 is made of a number (at least 4) sets of straight line segments. The placement of these four segements combine to resemble the arc that was fitted overall. However, none of the individual segments are consistent with the curvature of the overall fit.

In my view:

1) a cursory examination might suggest a glass with dimensions of #6 consistent with the curves on Folien 1,
2) however, from a more detailed look at the edges (both top and bottom) I conclude that it is highly improbable that the curves on Folien 1 were made by the edges of a single conical glass.

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